2026 Taiwan-Japan Joint Workshop on Inverse Problems and Related Topics

Date : 27 (Fri.) – 29 (Sun.) November, 2026
Venue : Lecture Room 32 (3rd floor), School of Science Main Building
Tsushima Campus, Okayama University, Japan

27 November (Fri.)

9:30–10:00 Registration

10:00–10:10 Opening Address
Jenn-Nan Wang and Masato Kimura

(Chair: Jenn-Nan Wang)

10:10–10:50 Shih-Hsien Yu (Academia Sinica)
Motions of a Stationary Shock Layer in the Presence of a Boundary

10:50–11:30 Hiroshi Fujiwara (Kyoto University)
Reconstruction of Three-Dimensional Fluid Density from Schlieren Images

Group Photo

(Chair: Takemi Shigeta)

13:00–13:40 Jhe-Kuan Su (Kyoto University)
Quantitative lower bounds for solutions to the Boltzmann equation in nonconvex domains

13:40–14:20 Meng-Kai Hong (National Central University)
Global Existence of Weak Solutions to the One-dimensional Transonic Flow.

(Chair: Meng-Kai Hong)

14:30–15:10 Julius Fergy Rabago (Kanazawa University)
Statistical Topological Gradient and Shape Optimization for Robust Metal–Semiconductor Contact Reconstruction

15:10–15:50 Pengwen Chen (National Tsing Hua University)
Phase retrieval algorithms and some reflections

(Chair: Yin-Tzer Shih)

16:00–16:40 Chiun-Chuan Chen (National Taiwan University)
Propagating Direction Near the Strong-Competition Borderline in the Two-Species Lotka–Volterra Model

16:40–17:20 Masato Kimura (Kanazawa University)
Hidden symmetry and positive-definiteness of P1-FEM-based spring constants in 2D/3D elasticity

28 November (Sat.)

(Chair: Pengwen Chen)

10:10–10:50 Kaede Watanabe (Tohoku University)
Ocean Wave Estimation Using HF Radar with Nonnegative Regularization

10:50–11:30 Yin-Tzer Shih (National Chung Hsing University)
A data-driven mathematical modeling framework for predicting HIV/HBV coinfection dynamics with machine learning

(Chair: Hisashi Morioka)

13:00–13:40 Pu-Zhao Kow (National Chengchi University)
Krylov methods for linear inverse problems in Hilbert spaces

13:40–14:20 Takemi Shigeta (Showa Pharmaceutical University)
Efficient numerical construction of the Dirichlet-to-Neumann map for time-harmonic wave propagation problems

(Chair: Masato Kimura)

14:30–15:10 Hisashi Morioka (Ehime University)
Quantum entanglement of scattered waves for 1D Schrödinger equation with the spin $s = 1/2$

15:10–15:50 Chun-Hsiung Hsia (National Taiwan University)
On the existence and regularity of weakly nonlinear stationary Boltzmann equations : a Fredholm alternative approach

(Chair: Hiroshi Fujiwara)

16:00–16:40 Takaaki Nishida (Kyoto University)
An incompressible limit of compressible viscous and heat-conducting fluid equation

16:40–17:20 Jenn-Nan Wang (National Taiwan University)
Inverse Born series based neural operators

17:20–17:30 Closing Address
Hiroshi Fujiwara

29 November (Sun.)

10:00–12:00 Personal Discussion @ Lecture Room 32

Organizers	Jenn-Nan Wang (National Taiwan University) Shih-Hsien Yu (Academia Sinica) Hiroshi Fujiwara (Kyoto University) Masato Kimura (Kanazawa University)
Local Organizers	Takashi Ohe (Okayama University of Science) Hiroshi Takase (Okayama University)

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Motions of a Stationary Shock Layer in the Presence of a Boundary

Yu, Shih-Hsien[1]

1. Institute of Mathematics, Academia Sinica, Taiwan
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Abstract

In this talk, we demonstrate that the presence of a boundary, through the coupling of nonlinearity and viscosity, induces motion in a stationary shock layer.

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Reconstruction of Three-Dimensional Fluid Density from Schlieren Images

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Abstract

This talk proposes a reconstruction procedure of the fluid density or temperature of a fluid by tomography in three dimensions. Assuming that visible light propagates through a transparent medium, we use Background-Oriented Schlieren (BOS) images [3,4], which has been developed rapidly in recent years. In BOS, the refraction of light rays by the fluid appears as displacements of a background pattern, and the density field is reconstructed from this displacement. Conventional tomography, such as X-ray tomography, relies on signals that travel straight. In contrast, measurement data in this study is the deflection of light [1,2], which is given by line integrals of the gradient of the refractive index, and thus the proposed reconstruction combines vector tomography and scalar tomography. In the presentation, we will formulate the forward problem and then discuss the reconstruction procedure. We will also demonstrate the effectiveness of the proposed method through numerical experiments.

References

[1] H. W. Liepmann and A. Roshko, Elements of gas dynamics, Dover Publications (2002)

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Quantitative lower bounds for solutions to the Boltzmann equation in nonconvex domains

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Abstract

The Boltzmann equation under consideration reads

$$\partial_t f(t,x,v) + v \cdot \nabla f(t,x,v) = Q[f,f](t,x,v),\ \forall (t,x,v) \in [0,T) \times \Omega \times \mathbb{R}^3, \tag{1}$$

$$f(0,x,v) = f_0(x,v),\ \forall (x,v) \in \Omega \times \mathbb{R}^3, \tag{2}$$

where $T > 0$. The problem of quantifying the positivity of the solutions to (1)-(2) with the boundary condition

$$f(x,v) = \alpha f(t,x,R(x,v)) + (1-\alpha)\left(\int_{w\cdot n(x)>0} f(t,x,w)(w\cdot n(x))dw\right)\frac{1}{2\pi T_B^2} e^{-\frac{|v|^2}{2T_B}} \tag{3}$$

has attracted considerable attention from many authors. Beyond interest in physics, the problem also plays a crucial role in kinetic theory. In this talk, we study continuous mild solutions to the Boltzmann equation in a bounded spatial domain, under either angular cutoff assumption or non-cutoff assumption. Without assuming convexity of the spatial domain, we establish a Maxwellian lower bound in the cutoff case, and a weaker-than-Maxwellian lower bound for the non-cutoff case. This extends the results of [1, 2], where the convexity of the domain was required.

References

[1] M. Briant, Instantaneous exponential lower bound for solutions to the Boltzmann equation with Maxwellian diffusion boundary conditions. *Kinet. Relat. Models.* 8 (2015), no. 2, 281–308.

[2] M. Briant, Instantaneous Filling of the Vacuum for the Full Boltzmann Equation in Convex Domains. *Archive for Rational Mechanics and Analysis.* 218 (2015), no. 2, 985–1041.

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Global Existence of Weak Solutions to the One-dimensional Transonic Flow.

John Meng-Kai Hong[1]

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Abstract

In this talk, we study the global weak solutions to the following two problems of one-dimensional hyperbolic resonant systems of balance laws:
(A) Initial-boundary value problem of Euler-Poisson equations for semiconductors,
(B) Generalized Riemann problem of isentropic Euler equations for nozzle flow.

For problem (A), we introduce a new version of generalized Glimm's scheme. The equations in problem (A) form a mixed-type PDE system, or a 2 by 2 hyperbolic system with global source. We transform this system into a 3 by 3 hyperbolic balance laws with local source. A new version of generalized Glimm's scheme is introduced to establish the global existence of weak solution for problem (A).

To problem (B), we treat the 2 by 2 Euler system as a 3 by 3 Isaacson-Temple system. The structure of the wave curves for the generalized Riemann solution is classified. Under the wave structure, we are able to construct the generic solution for the generalized Riemann problem. One of the interesting results we obtain is that the composite wave pattern can exist alone in 3 by 3 Isaacson-Temple systems.

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Statistical Topological Gradient and Shape Optimization for Robust Metal–Semiconductor Contact Reconstruction

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Abstract

The metal–semiconductor contact region in VLSI circuits governs the contact resistance, yet it is buried inside the device and can only be probed through boundary measurements [1]. We present a framework for reconstructing it that combines topology and shape optimization within the coupled complex boundary method [2], which uses a single complex Robin condition with a free parameter β. The topological gradient [3] first detects the number and approximate location of the contacts and provides the initial guess for shape optimization, which then refines their location, size, and geometry.

To make the topological stage robust to noise, we also present a *statistical topological gradient*, namely a stability estimate under noisy data and a central limit theorem for its empirical mean, which gives confidence zones for contact detection. Numerical experiments illustrate the reconstructions and the effect of β.

This is joint work with Lekbir Afraites (Sultan Moulay Slimane University, Morocco), Aissam Hadri (Ibn Zohr University, Morocco), and Mourad Hrizi (Monastir University, Tunisia).

References

[1] W. Loh, S. Swirhun, T. Schreyer, R. Swanson, and K. Saraswat, Modeling and measurement of contact resistances, *IEEE Trans. Electron Devices* **34** (1987) 512–524.

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Phase retrieval algorithms and some reflections

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Abstract

Phase retrieval is a fundamental inverse problem in physics. In experiments such as X-ray crystallography, coherent diffraction imaging, and scattering experiments, the photon (or particle) counts recorded by the detector are proportional to the intensity of the diffracted wave, that is, the squared magnitude of the Fourier transform of the object. As a result, the phase information cannot be measured directly. Recovering the signal from such phaseless data is known as the phase retrieval problem, which is nonlinear and often ill-posed. The presence of the Fourier transform in the measurement process makes the reconstruction considerably more difficult.

Convex optimization plays a central role in modern algorithms for this class of problems, particularly in improving the quality of the reconstruction. In this talk, I will introduce the main ideas behind these algorithms and, drawing on my own experience, discuss possible directions for future research through two examples.

The first example concerns the reconstruction of three-dimensional tomographic images from diffraction patterns. We adopt the tomographic framework based on the pseudopolar grid proposed by A. Averbuch et al. [1], develop a spectral method for inverting three-dimensional tomographic diffraction data, and carry out an extensive study of dose fractionation[2].

The second example concerns PhaseLift, a semidefinite programming approach proposed by E. J. Candès et al.[3], which we have successfully applied to reconstructing signals from small-angle scattering measurements[4]. In this part, we also discuss an interesting numerical phenomenon: the low numerical rank that arises when Gaussian measurement matrices are replaced by Fourier matrices.

References

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Propagating Direction Near the Strong-Competition Borderline in the Two-Species Lotka–Volterra Model

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Abstract

In this talk, we consider the propagation direction of bistable traveling waves in the two-species Lotka–Volterra competition-diffusion model under strong competition. From an ecological perspective, the sign of the wave speed is critical, as it determines which species eventually prevails when two competing species exhibit a wave-like spatial distribution. We present a minimax formulation to characterize the sign of the wave speed. Furthermore, we focus on a near-symmetric scenario where the intrinsic growth rates and inter-specific competition coefficients of both species are identical, leaving their diffusion rates as the sole source of asymmetry. This framework is motivated by the "Unity is not strength" conjecture proposed by Alzahrani, Davidson, and Dodds and further studied by Girardin and Nadin, which suggests that the species with the higher diffusion rate gains a competitive advantage, directly dictating the sign of the wave speed. While significant progress on this problem was recently made by Nakamura and Ogiwara, Kenne has recently fully validated this conjecture in an arXiv preprint. In this talk, we explore the subtle regime where inter-specific competition weakens as it approaches the strong-competition borderline. We provide more quantitative estimates for the sign of the wave speed in terms of competition strength and diffusion rates.

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Hidden symmetry and positive-definiteness of P1-FEM-based spring constants in 2D/3D elasticity

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Abstract

Spring-block systems are widely used in the discrete element method and related approaches, since they represent fracture in a natural way. However, the spring constants are often chosen empirically, and their consistency with continuum mechanics and their mathematical properties are not always clear.

We have proposed a framework that derives a spring-block system consistent with linear elasticity from the P1-finite element method (P1-FEM) [2, 3]. Interpreting the stiffness matrix as tensor-valued spring constants $K_{ij} \in \mathbb{R}^{d\times d}$ yields a spring-block system equivalent to the P1-FEM. For fracture models on such systems [3], the symmetry and positive-definiteness of each K_{ij} are essential, ensuring unique solvability and energy dissipation.

Based on [1], we present a unified 2D/3D theory of these properties. The symmetry $K_{ij} = K_{ij}^{\mathrm{T}}$ is a *hidden symmetry*: it does not follow from the symmetries of the elasticity tensor, but emerges from the P1 discretization. It was proved in 2D in [2], but the extension to 3D is nontrivial, since the number of tetrahedra sharing an edge is not fixed, unlike the 2D case. Using an identity for the gradient jumps of finite element functions, we give a unified proof for 2D and 3D.

For isotropic elasticity, we derive a necessary and sufficient condition for K_{ij} to be positive-definite, which depends on Poisson's ratio ν and the mesh geometry. For the equilateral triangular mesh in 2D, the condition reduces to $\nu < 1/4$ (plane strain) [2], whereas in 3D the geometric constraints of space-filling tetrahedra make it difficult to keep all K_{ij} positive-definite. Numerical experiments confirm these results.

References

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Ocean Wave Estimation Using HF Radar with Nonnegative Regularization

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Abstract

High-frequency (HF) radar is a system that transmits electromagnetic waves toward the ocean and receives the backscattered signals. These signals contain information about ocean surface waves and have been used to estimate ocean wave spectra, which characterize wave conditions at a given location. Accurate reconstruction of these spectra is important for estimating wave heights and other quantities used to describe sea states. However, further improvements in estimation accuracy are needed [1].

In this research, we propose and analyze a method for reconstructing ocean wave spectra based on nonnegative regularization. The proposed method incorporates the nonnegativity of wave spectra into the reconstruction process. We analyze the properties of the proposed method and introduce an optimization algorithm based on the proximal gradient method. We also evaluate its performance through numerical experiments, focusing on the accuracy of wave heights estimated from the reconstructed spectra. Numerical results demonstrate that the proposed method estimates wave heights more accurately than existing methods.

References

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A data-driven mathematical modeling framework for predicting HIV/HBV coinfection dynamics with machine learning

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Abstract

This study develops a fractional-order multi-stage HIV/HBV coinfection model using the Caputo fractional derivative to capture memory effects in disease transmission and progression. The model includes latent, asymptomatic, and symptomatic infection stages, as well as key epidemiological factors, including the effects of HBV vaccination, recovery processes among asymptomatic and fully infected HBV individuals, and the acquisition of secondary infections by exposed and asymptomatic individuals [1, 2]. The model's qualitative properties, such as positivity, boundedness, and sensitivity of key parameters, are analyzed to assess the biological feasibility of the system [1, 3]. The basic reproduction numbers for HIV and HBV are calculated, and a sensitivity analysis is conducted to identify the primary parameters affecting coinfection transmission. Real data from the Taiwan CDC [4] are used to fit the model. The fitting results demonstrate the model's effectiveness in handling the data. The coinfected HIV/HBV model is solved numerically using an artificial neural network optimized by the Levenberg-Marquardt algorithm. This study provides policymakers and epidemic managers with a reliable modeling tool for developing adaptable and sustainable strategies to control HIV/HBV coinfection transmission.

References

[1] M.A. Ullah, S.U. Saqib, Y.-T. Shih, *et al.* A data-driven mathematical modeling framework for predicting HIV/HBV coinfection dynamics using machine learning approaches, *AIMS Mathematics*, **11**(7), 22688–22723 (2026).

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Krylov methods for linear inverse problems in Hilbert spaces

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Abstract

In this talk, we present a general methodology for solving linear inverse problems modeled by injective compact operators on separable Hilbert spaces, using singular value decomposition combined with a carefully designed Krylov subspace method. Intuitively, a compact operator on a separable Hilbert space can be thought of as an infinite-dimensional matrix, it behaves like a matrix but in an infinite-dimensional setting.

A key advantage of our approach is that it requires no predefined low-rank approximation, we do not fix a Hilbert basis or truncate manually. Instead, the method automatically selects an appropriate basis and truncation level based on the residual. Once a suitable low-rank approximation is obtained, the linear inverse problem can be solved accordingly. To illustrate the approach, we apply it to the unique continuation problem for Herglotz wave functions as discussed in [1].

References

[1] Pu-Zhao Kow, Mikko Salo and Sen Zou, Increasing resolution and instability for linear inverse scattering problems. *J. Funct. Anal.* **289** (2025), issue 1, article number 110923, 37 pages. https://doi.org/10.1016/j.jfa.2025.110923

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Efficient numerical construction of the Dirichlet-to-Neumann map for time-harmonic wave propagation problems

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Abstract

Boundary value problems in unbounded domains arise in various applications, including wave scattering and acoustic analysis. The Dirichlet-to-Neumann (DtN) map provides an exact transparent boundary condition [1], allowing the original problem to be reduced to a bounded computational domain. However, its discretization generally leads to dense matrices, making efficient computation essential.

This talk presents efficient numerical methods for constructing and applying the DtN map based on the method of fundamental solutions (MFS), focusing on two- and three-dimensional exterior Helmholtz problems.

In two dimensions, equally spaced source and collocation points on concentric circles yield circulant matrices, enabling efficient computation using the fast Fourier transform (FFT). The resulting DtN map is incorporated into the finite element method (FEM) to solve exterior Helmholtz problems with inhomogeneous terms [3].

In three dimensions, source and collocation points are distributed using Gauss–Legendre nodes in latitude and equally spaced points in longitude. The resulting block-circulant structure is exploited by combining the FFT [2] with modal decomposition using associated Legendre functions. Approximate diagonalization in the modal basis further reduces the cost of applying the discrete DtN map. Numerical experiments demonstrate the accuracy and efficiency of the proposed approach.

Finally, potential applications to inverse problems, particularly those requiring repeated solutions of forward problems, are briefly discussed.

References

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Quantum entanglement of scattered waves for 1D Schrödinger equation with the spin $s = 1/2$

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Abstract

Quantum entanglement is a crucial concept in recent quantum technology. Mathematically the quantum entanglement is defined as follows. Let $\mathcal{H}$ and $\mathcal{K}$ be Hilbert spaces. The state $|\Psi\rangle \in \mathcal{H} \otimes \mathcal{K}$ is separable if there exist $|\psi\rangle \in \mathcal{H}$ and $|\phi\rangle \in \mathcal{K}$ such that $|\Psi\rangle = |\psi\rangle \otimes |\phi\rangle$. If $|\Psi\rangle$ is not separable, the state $|\Psi\rangle$ is entangled. By using the Schmidt decomposition ([1] and so on), we can distinguish entangled states with seperable states. Let $\{|\psi_j\rangle\}_j$ and $\{|\phi_k\rangle\}_k$ be the CONS of $\mathcal{H}$ and $\mathcal{K}$, respectively. For $|\Psi\rangle = \sum_{j,k} a_{j,k}|\psi_j\rangle \otimes |\phi_k\rangle \in \mathcal{H} \otimes \mathcal{K}$, we consider the matrix $A = [a_{j,k}]_{j,k}$. In view of the SVD, there exist unitary matrices such that $A = UDV^*$ where

$$D = \begin{bmatrix} D_r & O \\ O & O \end{bmatrix}, \quad D_r = \mathrm{diag}[\sigma_1, \dots, \sigma_r].$$

The Schmidt rank is defined by r as above. Then $|\Psi\rangle \in \mathcal{H} \otimes \mathcal{K}$ is separable if and only if $r = 1$.

In this talk, we consider the 1D matrix Schrödinger equation $(-D_x^2 + V)\psi = \lambda\psi$, $\lambda > 0$, for $\psi(x) = [\psi_{\downarrow\downarrow}(x), \psi_{\downarrow\uparrow}(x), \psi_{\uparrow\downarrow}(x), \psi_{\downarrow\downarrow}(x)]^{\mathsf{T}}$ where $V(x) = \sum_{j=0}^{3} v_j(x)\sigma_j \otimes \sigma_j$ with

$$\sigma_0 = I_2, \quad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

We assume that $v_j(x)$ is real-valued and $v_j \in \mathcal{S}(\mathbf{R})$. This model appears in the 2-body 1D scattering of particles with spin $s = 1/2$. Taking a incident plane wave $\psi^{(0)}$, we can construct a scattered wave by solving the Lipmann-Schwinger equation $\psi = \psi^{(0)} - R_0(\lambda + i0)V\psi$ with $R_0(z) = (-D_x^2 - z)^{-1}$. Namely, ψ has the asymptotic behavior

$$\psi(x) \sim \begin{cases} e^{i\sqrt{\lambda}x}|\tau(\lambda)\rangle, & x \to \infty, \\ e^{i\sqrt{\lambda}x}|e_{s_1}\rangle \otimes |e_{s_2}\rangle + e^{-i\sqrt{\lambda}x}|\rho(\lambda)\rangle, & x \to -\infty, \end{cases} \qquad |\rho(\lambda)\rangle, |\tau(\lambda)\rangle \in \mathbf{C}^2 \otimes \mathbf{C}^2.$$

Even if $\psi^{(0)}$ is separable, $|\rho(\lambda)\rangle$ and $|\tau(\lambda)\rangle$ may be entangled. Some numerical examples are going to be disclosed in this talk.

References

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On the existence and regularity of weakly nonlinear stationary Boltzmann equations : a Fredholm alternative approach

Chun-Hsiung Hsia

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Abstract

This is joint work with I-Kun Chen and Daisuke Kawagoe. The celebrated Fredholm alternative theorem works for the setting of identity compact operators. This idea has been widely used to solve linear partial differential equations. In this talk, we demonstrate a generalized Fredholm theory in the setting of identity power compact operators, which was suggested in 1987 by Cercignani and Palczewski to solve the existence of the stationary Boltzmann equation in a slab domain. We carry out the detailed analysis based on this generalized Fredholm theory to prove the existence theory of the stationary Boltzmann equation in bounded three-dimensional convex domains. To prove that the integral form of the linearized Boltzmann equation satisfies the identity power compact setting requires the regularizing effect of the solution operators. Once the existence and regularity theories for the linear case are established, with suitable bilinear estimates, the nonlinear existence theory is accomplished.

References

[1] I-Kun Chen, Chun-Hsiung Hsia, Daisuke Kawagoe, On the existence and regularity of weakly nonlinear stationary Boltzmann equations: a Fredholm alternative approach, Journal of Differential Equations, Volume 484, 15 December 2026, 114775.

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An incompressible limit of compressible viscous and heat-conducting fluid equation

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Abstract

The system of compressible viscous heat-conducting fluid equations is the following for the density ρ , velocity $\boldsymbol{u}$ and temperature T :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 ,$$
$$\rho \frac{\partial \boldsymbol{u}}{\partial t} - \mu \Delta \boldsymbol{u} - \lambda \nabla (\nabla \cdot \boldsymbol{u}) + \nabla p + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = 0 ,$$
$$\rho \frac{\partial T}{\partial t} - \kappa \Delta T + p \nabla \cdot \boldsymbol{u} + \rho \boldsymbol{u} \cdot \nabla T = \mu |\nabla \boldsymbol{u}|^2 + \lambda (\nabla \cdot \boldsymbol{u})^2 ,$$

where the equation of state $p = R\rho T$ and the internal energy $e = c_V T$ are assumed ($R = (\gamma - 1) c_v$) .

We use a usual scaling for the density, velocity, temperature etc. :

$\frac{\partial}{\partial t} \to \epsilon \frac{\partial}{\partial t}$, $\rho \to 1 + \epsilon \rho$, $\boldsymbol{u} \to \epsilon \boldsymbol{u}$, $T \to 1 + \epsilon \theta$, $\mu \to \epsilon \mu$, $\lambda \to \epsilon \lambda$, $\kappa \to \epsilon \kappa$.

The system is reduced to the following.

$$\frac{\partial \rho}{\partial t} + \frac{1}{\epsilon} \nabla \cdot \boldsymbol{u} + \nabla \cdot (\rho \boldsymbol{u}) = 0 ,$$
$$(1 + \epsilon \rho) \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right) - \mu \Delta \boldsymbol{u} - \lambda \nabla (\nabla \cdot \boldsymbol{u}) + \frac{1}{\epsilon} \nabla p = 0 ,$$
$$\frac{(1 + \epsilon \rho)}{\gamma - 1} \left(\frac{\partial \theta}{\partial t} + \boldsymbol{u} \cdot \nabla \theta \right) - \kappa \Delta \theta + \frac{1}{\epsilon} (1 + \epsilon p) \nabla \cdot \boldsymbol{u} = \mu |\nabla \boldsymbol{u}|^2 + \lambda (\nabla \cdot \boldsymbol{u})^2 ,$$

where $p = \rho + \theta + \epsilon \rho \theta$.

We consider the initial boundary value problem for this system with the slip boundary condition in the cuboid domain Ω , whose unknown functions are extended as odd or even periodic ones. Our aim is the following.

The solutions exist globally in time for the small initial data in $H^s(\Omega)$ and for small $\epsilon > 0$. Then we get the incompressible Navier-Stokes equations as a limit of ϵ tending to zero .

$$\nabla \cdot \boldsymbol{u} = 0 ,$$
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \mu \Delta \boldsymbol{u} + \nabla p = 0 ,$$
$$\frac{1}{\gamma - 1} \left(\frac{\partial \theta}{\partial t} + \boldsymbol{u} \cdot \nabla \theta \right) - \kappa \Delta \theta = 0 .$$

References

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Inverse Born series based neural operators

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Abstract

The inverse Born series provides a general perturbative framework for representing nonlinear inverse maps between infinite-dimensional function spaces and has found numerous applications in inverse problems governed by partial differential equations and integral equations. Motivated by its operator-theoretic structure, we develop a systematic framework for constructing neural operators that approximate the operator expansions arising in the inverse Born series. Our approach combines the analytical representation of the inverse Born series with the expressive power of neural operators, yielding data-driven approximations of the nonlinear inverse map while preserving the underlying operator structure.

The proposed framework is applicable to a broad class of inverse problems and is presented in a general functional-analytic setting. To demonstrate its practical performance, we consider two representative examples: inverse scattering and the Calderón (electrical impedance tomography) problem. Numerical experiments show that the constructed neural operators accurately approximate the inverse Born expansions and produce high-quality reconstructions across a range of test cases. These results indicate that the proposed methodology provides an effective and computationally efficient approach for learning nonlinear inverse operators and suggests a promising direction for integrating classical operator expansions with modern neural operator architectures. This talk is based on a joint work with A. Titi and J. Schotland.